PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

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Phil. Trans. R. Soc. Lond. A 1980 **297**, 629-640
doi: 10.1098/rsta.1980.0238

Phil. Trans. R. Soc. Lond. A **297**, 629–640 (1980) [629]
Printed in Great Britain

Propagation of solar particles in the interplanetary medium

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Our view of the spectrum and time dependence of the energetic particles accelerated during flare events is distorted by the diffusion and energy changes that take place during the propagation of the particles through the interplanetary medium. We describe theoretical attempts to calculate the transport coefficients in space and energy and to represent the observed time dependence and pitch angle distributions both near the Earth and on distant space probes.

Particular attention is given to the interplanetary acceleration processes that are thought to occur, both in the neighbourhood of interplanetary shocks and generally throughout the interplanetary medium. Current experimental evidence on these effects are reviewed. Revised particle transport equations which take into account the acceleration are discussed.

1. Introduction

This contribution is related in particular to Keppler's (this symposium) review of the experimental situation concerning the composition and spectra of energetic solar particles and the lectures on the overall and disturbed microstructural states of the solar wind medium given respectively by Hedgecock (unpublished) and Schwartz & Roxburgh (this symposium). We shall not be concerned with the initial acceleration of energetic particles in solar flares but rather with the interaction of the particles with the interplanetary medium once they have left the inner solar corona where closed magnetic field configurations exist and dominate. The physics of interest to us here is the interaction of these particles with the solar wind plasma, leading to problems of diffusion in position and velocity space and in the definition of the wave structure in the interplanetary medium. Thus we both exploit the nearby spatial medium to investigate some fundamental plasma physical problems and also endeavour to provide a means both for investigating the dynamic evolution of this medium and for extrapolating knowledge of the solar particle spectra back to their source values. Our emphasis will be on the new situation regarding the particle transport problem that arises because of the recent experimental evidence for significant interplanetary acceleration of the solar energetic particles.

2. Energetic particle transport without interplanetary acceleration

Setting aside special processes occurring at interplanetary shock fronts, the general, pre-1976, view of solar particle transport was as follows. A solar flare releases ions with $E \lesssim 100$ keV up to possibly as much as *ca.* 10 GeV, the composition being mainly protons but with species up to $Z = 44$ identified. This release occurs in a time scale usually short and of the order of minutes, but probably not more than 1 or 2 h in extent except in some possible exceptional cases. It results in the propagation of particles approximately along the Archimedes spiral field lines of the solar wind. The reason for this mode of propagation is that the cyclotron radii of the particles are small compared with 1 AU† and the guiding centres attempt to follow the field lines. Resonant wave–particle interactions in which low frequency Alfvén waves in the medium spatially match in wavelength the distance travelled by the particles in one cyclotron revolution cause diffusion in pitch angle and hence in the propagation parallel to the field. Thus the time profile of the flux arrival at the Earth initially resembles that expected in a one-dimensional diffusion situation with a single point in time injection profile. Later on, the effect of the solar wind convection of the Alfvén wave scattering centres becomes apparent and the solar particles tend to equilibrate with the outward moving solar wind frame of reference. In particular, the direction of the particle anisotropy moves around from *ca.* 45° west of the Earth–Sun line (the most probable Archimedes spiral direction at the Earth's orbit) to a more radial direction. At a time when the diffusion gradient is zero, the anisotropy is purely determined by the Compton–Getting effect which is a consequence of transforming an isotropic flux distribution in the solar wind reference frame to the stationary (terrestrial) frame (Gleeson & Axford 1968). This transformation causes anisotropy by crowding together the distribution function flux vectors in the direction of solar wind motion and by increasing the energy of each particle to bring it into a higher differential energy range and thus, because of the negative exponent of the energy spectrum, increases the laboratory frame flux.

At late times in a solar energetic particle event, the effects of diffusion, convective sweeping and the net deceleration of particles as they collide with the receding scattering centres of the expanding solar wind puts the position of the peak particle flux beyond the Earth's orbit. Hence the anisotropy due to the diffusion gradient is inwards; but when compounded with the Compton–Getting anisotropy the net result is an outward flow from east of the Earth–Sun line. McCracken *et al.* (1971) discuss experimental evidence for this description while Ng & Gleeson (1971) provided a theoretical formalism.

For motion along a magnetic flux tube connected to the flare site with negligible perpendicular diffusion, as appears to be the situation in practice, the transport equation may be derived from the continuity equation in time, position and energy space (see, for example, Gleeson & Axford 1967):

$$\frac{\partial N}{\partial t}+\frac{1}{r^2}\frac{\partial}{\partial r}r^2S = \frac{\partial}{\partial T}\left(\frac{\partial T}{\partial t}\right)N. \tag{1}$$

Spherical symmetry is assumed, N is the differential number density at kinetic energy T, and S is the streaming in the radial direction;

$$S = CVN - K_r\,\partial N/\partial r,$$

where V is solar wind velocity, K_r is the radial component of the parallel to the field spatial diffusion coefficient, C is the Compton–Getting factor, $C = 1-\frac{1}{3}N^{-1}\,\partial(\alpha TN)/\partial T$ where $\alpha = (T+2E_0)/(T+E_0)$ for particle rest energy E_0. The deceleration term $N\,\partial T/\partial t = V\,\partial(\frac{1}{3}\alpha TN)/\partial r$, which may be thought of as the work done by the cosmic ray gas on the solar wind or Vd (pressure) (see, for example, Fisk 1974).

Equation (1) reduces to the Fokker–Planck first derived by Parker (1965):

$$\frac{\partial N}{\partial t}+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2VN - r^2K_r\frac{\partial N}{\partial r}\right) = \frac{1}{3r^2}\frac{\partial}{\partial r}(r^2V)\frac{\partial}{\partial T}(\alpha TN). \tag{2}$$

† 1 AU ≈ 15×10^{10} m.

3. The evidence for acceleration

It has been known for a long time that a description of solar particle transport based on solutions of (2) for a ∂-function in time injection cannot adequately describe at least one class of special events. This class is the recurrent increases in the 1–10 MeV energy range found by Bryant *et al.* (1965), which appear again after one or more solar rotations. These were first though to be due to continuous injection at the Sun, but observations by Barnes & Simpson (1976) on Pioneer 10 and 11 out to 8 AU showed various series of recurrent increases in pairs associated with the leading and trailing edges of well identified fast solar wind streams. This correlation appears beyond *ca.* 2.5 AU where the fast stream interaction with the ambient solar wind develops into a forward and reverse shock pair. Barnes & Simpson examined the steepening of the proton spectral index near the leading shock and the increase in proton:helium abundance ratio and concluded from the lack of radial dependence to these effects and maintenance of the corotating time-profile with distance that acceleration is taking place at the shock interfaces and/or in the enhanced magnetic fluctuations of the fast stream.

Another set of experimental evidence pointing towards interplanetary acceleration is that provided by Van Hollebeke *et al.* (1978). These workers examined the maximum particle intensity found in recurring streams relative to that observed on the same flux tube at 1 AU and found positive gradients of *ca.* 100%/AU from 1–5 AU. However, beyond that point the relative intensity fell with distance from the Sun. No significant heliolatitude effects were found that could confuse the deduction that particle acceleration occurs out to 5 or 6 AU, associated either with the enhanced fluctuations or the shocks belonging to the fast solar wind streams.

Somewhat more controversial is the evidence put forward by Marshall & Stone (1978) that excluding the times of prompt solar proton events, 1.3–2.3 MeV protons exhibit a net inward streaming at 1 AU, as seen in the rest frame. This apparent direct evidence of a significant source beyond the Earth's orbit is disputed by Zwickl & Roelof (1979), who found in the 0.3–0.5 MeV proton energy range evidence only for the $\boldsymbol{E} \times \boldsymbol{B}$ drift motion perpendicular to $\boldsymbol{B}$ for non-impulsive events. An absence of parallel to $\boldsymbol{B}$ field streaming is implied by the data analysis of these latter authors.

Further evidence of a general interplanetary acceleration mechanism is provided by Fisk *et al.* (1974) and Fisk (1976*a*). These authors have sought an explanation of the anomalously high fluxes of He, N, O and Ne particles appearing in the near Earth cosmic ray spectrum below energies of 10 MeV/nucleon (McDonald *et al.* 1974; Garcia-Munoz & Simpson 1973). It is suggested that interstellar neutral particles that enter the solar cavity become singly ionized as a result of solar ultraviolet interactions and charge exchange with solar wind ions and then take up the solar wind motion. Some small fraction of the particles are then slowly accelerated to 10 MeV nucleon at which point the singly charged He, N, O and Ne have such high rigidities that a significant fraction diffuse back into the inner solar region, unlike the low rigidity, fully ionized H. Fisk (1976*a*) demonstrated that an acceleration rate with a diffusion coefficient in kinetic energy, D_{TT}, given by $D_{TT} \approx 5 \times 10^{-7}\, T\,(\mathrm{MeV})^2\,\mathrm{s}^{-1}$, where T is in megaelectronvolts, can satisfy the cosmic ray spectral observations.

4. Shock acceleration

Experimental evidence for short timescale solar proton intensity enhancements associated with interplanetary shock waves (shock spikes) has been reported many times (e.g. by Van Allen & Ness 1967). The theory of the acceleration process is reviewed by, for example, Armstrong *et al.* (1977). An interplanetary shock wave is most efficient at giving energy to particles if the angle between the shock normal and the magnetic field is less than 10°. If this angle, β, is between 0 and 5°, the shocks are called 'perpendicular', while if $5° \leqslant \beta \leqslant 10°$, they are known as 'small angle oblique'.

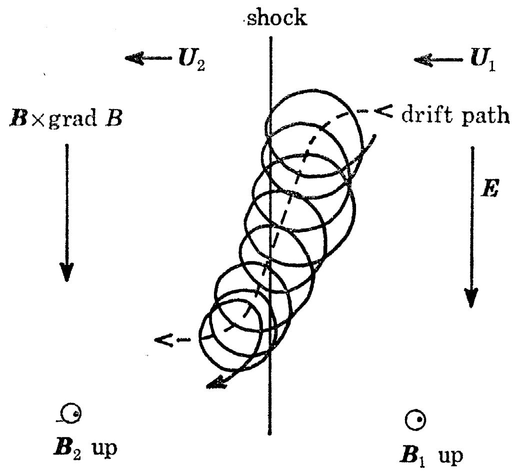

Figure 1. Drift of a positively charged particle through a perpendicular shock due to the influence of the electric field drift force and the magnetic field strength change. Parameters relative to the shock frame of reference are shown with B_1, B_2 and the shock surface perpendicular to the paper.

(a) *Perpendicular shocks*

All particles incident on a perpendicular shock cross the boundary. The energenization is demonstrated in figure 1, which represents the rest frame of the shock. In this frame, upstream field $\boldsymbol{B}_1$ (out of paper) flows into the shock with velocity u_1 and downstream field $\boldsymbol{B}_2$ (out of paper) flows away with velocity u_2. An electric field $|E| = -u_1 B_1 = -u_2 B_2$ therefore exists in the direction shown, parallel to the shock surface. An $\boldsymbol{E} \times \boldsymbol{B}$ drift takes particles through the shock from upstream to downstream while the magnetic forces, qualitatively approximated by a $\boldsymbol{B} \times \text{grad}\, B$ drift move the particles in the $\boldsymbol{E}$ field direction, parallel to the surface. Hence energenization in a plane perpendicular to $\boldsymbol{B}$ occurs and the particle anisotropy is expected to exist in this plane.

(b) *Small angle oblique shocks*

Figure 2 represents the shock front moving in the rest frame with velocity V_s. Although particles with small pitch angles may be transmitted, as in the previous case, other particles at high pitch angles and certain phase angles may be reflected by acquiring a $\boldsymbol{v}_\parallel$ in the pre-shock régime away from the surface which allows them to outrun the advancing shock. The process of reflection is illustrated in the figure where we plot $\boldsymbol{v}$, the velocity vector of a particle which has just crossed from the upstream régime. $\boldsymbol{v}_\parallel$ represents the parallel component of this velocity projected onto the upstream field direction while $\boldsymbol{v}'_\parallel$ is the same parameter projected onto the downstream field direction. Suppose subsequent motion around the field line $\boldsymbol{B}_2$

results in an intersection with the shock surface with velocity vector $\boldsymbol{v}'$. The particle will emerge into the upstream régime with $\boldsymbol{v}''_{\parallel}$, considerably less than $\boldsymbol{v}_{\parallel}$, in the towards shock direction. Multiple crossings can therefore result in complete reflexion.

Computations of the energy change due to the electric field show typical ratios T (final)/T (initial) ≈ 3 for $(|v|/|V_s|) \approx 10$, and $|\boldsymbol{B}\text{ (normal)}|/|\boldsymbol{B}| \approx 0.1$ in the upstream régime.

Experimentally there is a very clear association of small oblique shocks with the occurrence of shock spikes. However, the expected anisotropy at large angles to $\boldsymbol{B}$ is not clearly evident. Rather the streaming is along the field lines, usually away from the shock but sometimes even towards it (Balogh 1977).

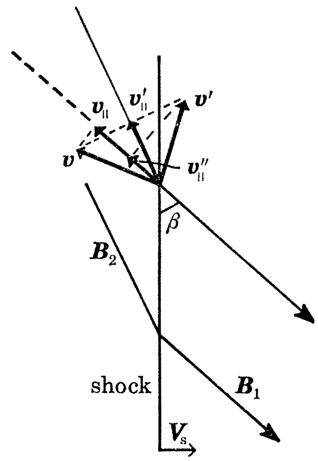

Figure 2. Velocity vector of an energetic particle during successive crossings of a small angle oblique shock, seen in the stationary reference frame: $\boldsymbol{v}$ is the velocity at the first crossing with $\boldsymbol{v}_{\parallel}$ and $\boldsymbol{v}'_{\parallel}$ resolved respectively on to the upstream and downstream region field directions; $\boldsymbol{v}'$ is the velocity at the re-emergence into the upstream region with $\boldsymbol{v}''_{\parallel}$ resolved on to the upstream field direction.

5. General interplanetary acceleration

For the remainder of this review I concentrate on estimating the magnitude of the acceleration that may happen throughout the interplanetary medium and predicting the effects of this acceleration on the solar proton transport. I start by rewriting the Fokker–Planck energetic particle transport equation with the inclusion of an energy gain term. Fisk (1976*a*) shows that the divergence term for the distribution function in momentum space,

$$\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 D_{pp}\frac{\partial t}{\partial p}\right),$$

can be transformed to yield the last two terms on the right-hand side of the following Fokker–Planck, written in terms of kinetic energy T:

$$\frac{\partial N}{\partial r} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 K_r \frac{\partial N}{\partial r}\right) + \frac{2}{3}\frac{V}{r}\frac{\partial}{\partial T}(\alpha T N) - \frac{V}{r^2}\frac{\partial}{\partial r}(r^2 N) + \frac{\partial}{\partial T}\left(D_{TT}\frac{\partial N}{\partial T}\right) - \frac{\partial}{\partial T}\left(\frac{D_{TT}}{2T}N\right). \quad (3)$$

Tverskoy (1967) put forward the initial theoretical discussion of the two types of Alfvén turbulence–particle interaction which lead to acceleration in the solar wind. These are adiabatic scattering by long wavelength changes in $\boldsymbol{B}_{11}$, which are very similar to the original Fermi process, and cyclotron resonance scattering, when the particle makes one Larmor revolution in the wavelength of a transverse Alfvén wave. Fisk (1976*a*) investigated the requirements of this second process where $D_{TT} \approx V_A^2 T^2/K_\parallel$, where V_A is the Alfvén speed and $K_\parallel$ is the parallel diffusion coefficient. For example, to obtain a factor 10 increase in the 1 MeV solar proton intensity in a corotating stream between 1 and 3 AU from the Sun, it is found that $D_{TT} \approx 1.4 \times 10^{-6}\, T^{\frac{3}{2}}$ (MeV)^2 s^{-1} or $K_\parallel \approx 1.8 \times 10^{19}$ cm^{-2} s^{-1} or $\lambda_\parallel \approx 3 \times 10^{-3}$ AU. This last value, for the parallel diffusion mean free path is, clearly contradicted by solar proton time profile data, being at least an order of magnitude too small. Hence the cyclotron resonance acceleration is too inefficient. Fisk (1976*b*) performed a detailed quasi-linear theoretical computation of the long wavelength $\delta \boldsymbol{B}$ effect, which he called transit time damping, and obtained a more satisfactory acceleration rate.

In outline, Fisk's work may be explained in the following physical manner: a particle of momentum p interacting with a wave of phase speed v_{wave} suffers a momentum change Δp and a speed change $\Delta\omega_\parallel$ in the reference frame moving with v_{wave}:

$$\Delta p/p = v_{\text{wave}}\, \Delta\omega_\parallel/\omega^2; \quad |\omega| = \text{constant}.$$

We have used the conservation of energy in the wave frame. $\Delta\omega_\parallel$ is determined by the conservation of the first adiabatic invariant at pitch angles θ near 90°, there being no electric field in the wave frame:

$$\frac{d\omega_\parallel}{dt} = -\frac{\omega^2 \sin^2\theta}{2B_0}\frac{\partial B}{\partial z}.$$

$\partial B/\partial z$ is related to a wavenumber $k_\parallel = 1/\lambda_z'$ by

$$\frac{1}{B_0}\frac{\partial B}{\partial t} = \frac{\eta}{\lambda_z'},$$

where η is the average fractional strength of fluctuations.

Defining a wavenumber $k = 1/\lambda'$, estimating $k_\perp$ as $(1/\lambda_x^2 + 1/\lambda_y^2)^{\frac{1}{2}}$ where λ_x^2 and λ_y^2 are correlation lengths in the x and y (perpendicular to $\boldsymbol{B}$) directions, and putting $k_\parallel = 1/\lambda_{z'}$ we find the phase speed u and propagation vector $\boldsymbol{k}$ are at an angle

$$\theta = \operatorname{arcsec}\left\{\lambda_{z'}\left(\frac{1}{\lambda_x^2} + \frac{1}{\lambda_y^2}\right)^{\frac{1}{2}}\right\} \quad \text{to} \quad \boldsymbol{B}_0.$$

The fast magnetosonic mode wavefront responsible for the adiabatic reflexion moves in the $\boldsymbol{B}_0(z)$ direction at a phase speed $u_\parallel = u \sec\theta$. The interaction is with $\lambda_z' \gg \lambda_z$ (correlation length) but takes place only for time $\Delta t \approx \lambda_z/|\cos\theta|\omega$.

Putting all these expressions together yields

$$\frac{D_{pp}}{p^2} = \frac{(\Delta p)^2}{p^2 \Delta t} = \frac{u^2 \sin^4\theta}{4|\cos\theta|\omega}\,\eta^2 \lambda_z \left(\frac{1}{\lambda_x^2} + \frac{1}{\lambda_y^2}\right).$$

This expression corresponds, to within a small numerical factor, to Fisk's case when the spectrum of turbulence in the interplanetary waves is proportional to k^{-2} or steeper and exhibits the sin^4 (pitch angle) factor. Hence pitch angle scattering on a time scale less than the acceleration time scale is required to keep the latter process efficient.

Fisk considered a less steep k dependence more representative of interplanetary conditions and obtained the numerical value

$$D_{TT} = 5 \times 10^{-7}\, T\, (\text{MeV})^2\, \text{s}^{-1}$$

consistent with the requirements of the mechanism to produce the anomalous He, etc., cosmic ray spectrum.

Moussas & Quenby (1978) have approached the problem from the viewpoint of numerical experiments, by using satellite data as a direct input. These authors had already obtained improved estimates of the spatial diffusion coefficient ($K_{\parallel}$) by numerical integrations and they have now added the electric field to the code. The basis of the method is to obtain the mean magnetic field direction from high time resolution continuous satellite data and then set up a slab model for the field, each plane perpendicular to the mean field and representing the three-dimensional measured $\boldsymbol{B}$ in that slab. Slabs are *ca.* 0.01 particle gyroradius in width. To obtain $K_{\parallel}$, sample trajectories are integrated with an input distribution function, isotropic in phase angle but at a unique μ_0 (cosine pitch angle). Particles are removed at boundaries μ_1 and μ_2 either side of μ_0 and a steady state $f(\mu)$ distribution obtained. Hence by knowing the particle currents $J_{1,2}$ at the boundaries, the pitch angle diffusion coefficient $D(\mu)$ is found from

$$D(\mu)\ \partial f/\partial\mu = -J_{1,2}.$$

$K_{\parallel}$ is then obtained by the Hasselmann & Wibberenz (1970) expression

$$K_{\parallel} = \tfrac{1}{4}\omega^2 \int_{-1}^{1} \left(\int_{0}^{\mu_1} \left(\frac{1-\mu^2}{D(\mu)} \right) \mathrm{d}\mu \right) \mu_1 d\mu_1.$$

Moussas & Quenby obtained $\lambda_{\parallel} = 0.031$ AU for 100 MeV particles ($K_{\parallel} = \frac{1}{3}\omega\lambda_{\parallel}$).

To obtain the acceleration, the electric field in the rest frame, $\boldsymbol{E} = -\boldsymbol{V}(\text{wind}) \times \boldsymbol{B}$ was computed from 5 min plasma data measured on the same satellite HEOS 2 (plasma data, Munich group; magnetometer data, Imperial College group) and the energy gain $\Delta T = e\boldsymbol{E}\cdot\boldsymbol{\omega}\Delta t$ was followed in the rest frame. Various ways of deriving D_{TT} were tried. Sample trajectories were followed and $D_{TT} = \langle \Delta E^2/2\tau \rangle$ was computed for various intervals τ, but this method suffered from very large errors and also interference from the periodic energy gains and losses with the cyclotron period. More satisfactory was essentially to repeat the $D(\mu)$ method by injecting particles at E_0 and building up the steady-state distribution defined by $f(E)$ against E with mean values of E_1 and E_2 where particles were removed from the field region. Preliminary results of Moussas & Quenby suggest $D_{TT} = 4 \times 10^{-7}\ T\,(\text{MeV})^2\ \text{s}^{-1}$ around $E_0 = 10$ MeV, in reasonable agreement with Fisk's estimate.

6. Transport equation solutions with general acceleration

Having obtained some theoretical and numerical experiment estimates of the general acceleration term in the interplanetary medium at typical solar proton energies, it is now important to check if the observed solar proton prompt event time profiles are consistent with this additional physical effect. In the past, the observed time of maximum at the Earth's orbit has been taken as a sensitive measure of the mean diffusion coefficient between 0 and 1 AU. The presence of interplanetary acceleration will slow the rise of the particle flux to maximum, thus causing an underestimate of the parallel diffusion mean free path, $\lambda_{\parallel}$.

To illustrate the effect of acceleration, Cecchini *et al.* (1980) have numerically solved the transport equation (3) including the acceleration term under the special assumptions that the input solar spectrum is of a power law shape and that $D_{TT} \propto T^2$ rather than $\propto T$. However, in this second assumption, the numerical value of D_{TT} at 10 MeV, the input energy for the calculations, was adjusted to correspond to that given by the theoretical estimates discussed in §5. In the computations, the value of the spatial diffusion coefficient at 1 AU is fixed by the value given by the Moussas–Quenby (1978) work but the radial dependence is allowed to be a free parameter, subject to $K_r = K_0 r^b$. Some idea of the radial dependence expected of D_{TT} is afforded by Fisk's theory which shows that $D_{TT} \propto \langle \delta B^2 \rangle / B^2$. Experiment suggests that $\langle \delta B \rangle \propto B$ far out while theory suggests $\delta B \propto r^{-\frac{3}{2}}$ for Alfvénic fluctuations. Sari (1977) puts forward experimental evidence that longitudinal interplanetary waves could be finite amplitude, non-transverse Alfvén waves ($|\boldsymbol{B}| = \text{const.}$, $B_z \neq \text{const.}$). Since $B \propto r^{-2}$ at $r \ll 1$ AU and $B \propto r^{-1}$ at $r \gg 1$ AU, D_{TT} showing a peak in importance at *ca.* 1 AU or being independent of r are two reasonable behaviour choices.

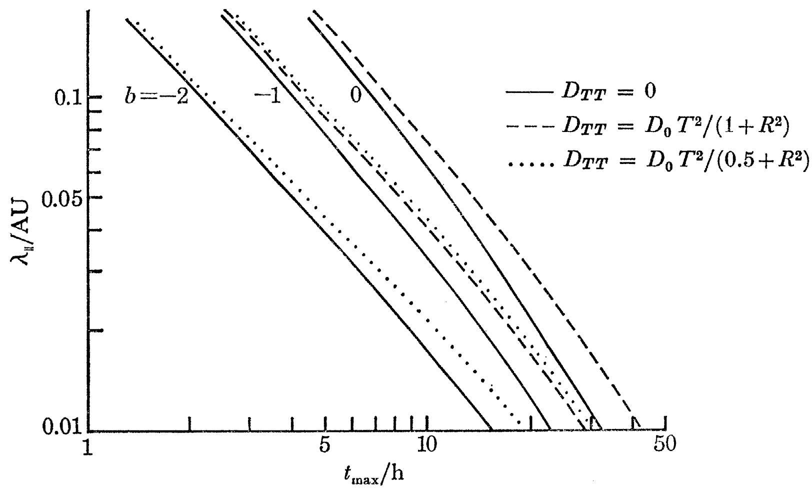

FIGURE 3. Time of maximum particle flux at 1 AU plotted against parallel diffusion mean free path, $\lambda_\parallel$, at 1 AU for various radial dependencies of the spatial diffusion coefficient, $K_r = K_0 r^b$ and values of energy diffusion coefficient, D_{TT}. $D_0 = 10^{-6} T^2$ MeV^2 s^{-1} and R is in AU.

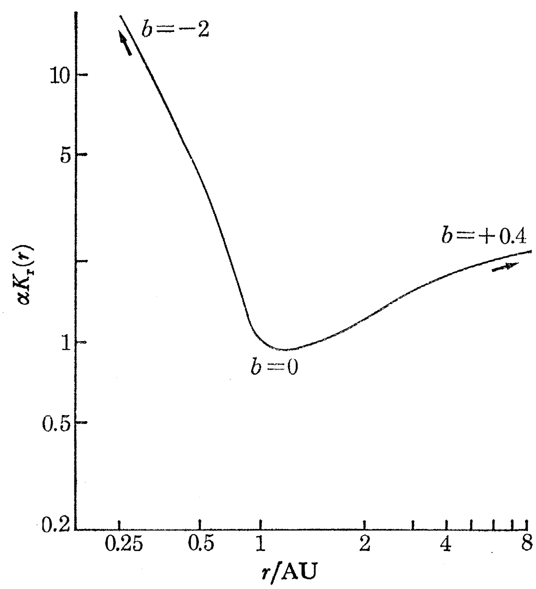

FIGURE 4. Form of radial dependence of the radial component of the spatial diffusion coefficient, $K_r(r)$, adopted for computations leading to the predictions shown in figures 5–8.

Figure 3 shows the computed time to maximum solar proton (10 MeV) flux against $\lambda_{\parallel}$ at 1 AU for $b = 0, -1, 2$ and

$$D_{TT} = 0$$

and
$$D_{TT} = \frac{10^{-6}\ T^2(\text{MeV})^2\ \text{s}^{-1}}{1.00+R^2} \quad \text{or} \quad \frac{10^{-6}\ T^2(\text{MeV})^2\ \text{s}^{-1}}{0.5+R^2},$$

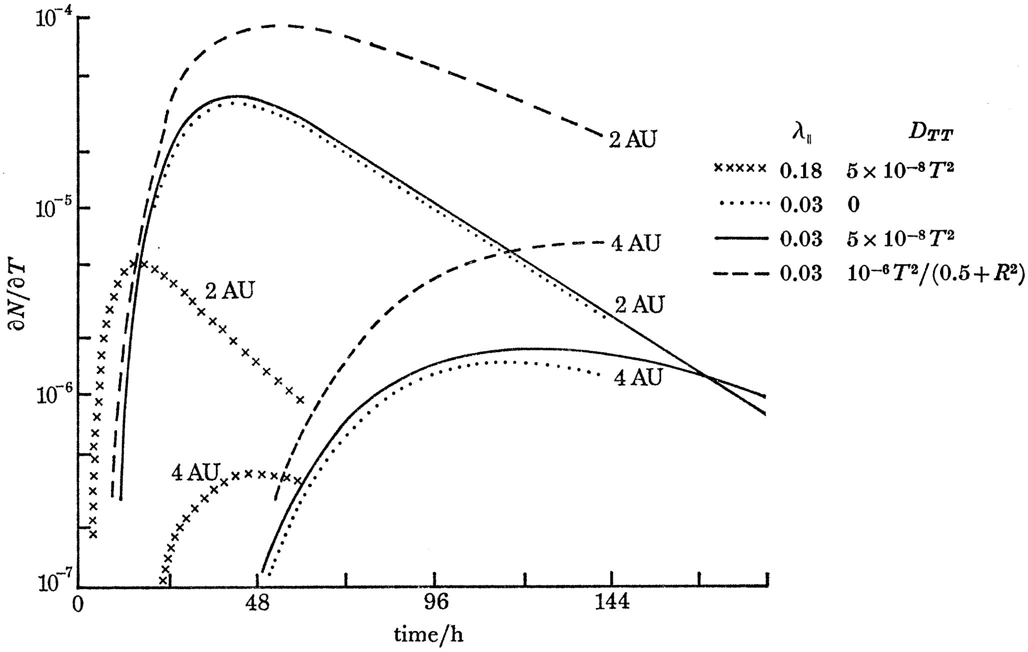

FIGURE 5. Predicted solar proton time profiles $(\partial N/\partial T)$ (t) at 2 and 4 AU for various values of the spatial diffusion mean free path, $\lambda_{\parallel}$, at 1 AU and of the energy diffusion coefficient, D_{TT}. $\lambda_{\parallel}$ is in AU; D_{TT} in $(\text{MeV})^2\ \text{s}^{-1}$.

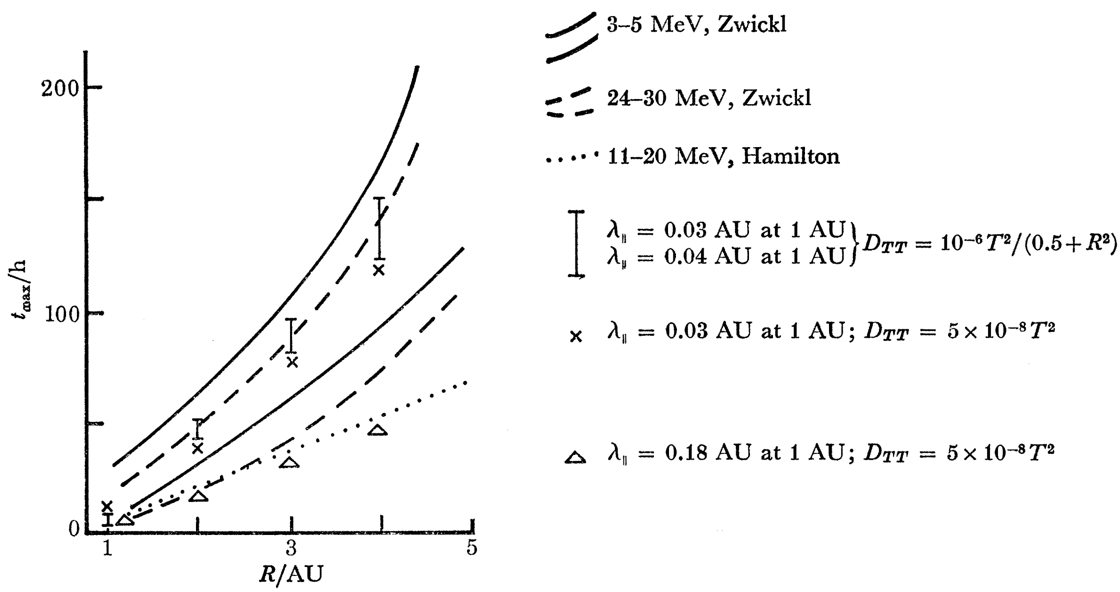

FIGURE 6. Pioneer 10 and 11 data on the time of maximum of the solar proton flux measured at different radial distances by Zwickl & Webber (1977) and Hamilton (1977). Theoretical predictions for various model parameters as shown are also plotted. Units as for figure 5.

where R is in AU. The value of D_{TT} adopted is actually 25 times larger than that given by the Moussas–Quenby numerical experiment at 10 MeV. However, the effect of such values of the acceleration does not appear to be dramatic, and for a given value of $\lambda_{\parallel}$ and b it would be hard to distinguish experimentally the different times of maximum with and without acceleration. The situation with regard to b is different and it is clear that the only way to obtain a time of maximum of about 10 h or less, as required by observations, consistent with $\lambda_{\parallel} = 0.03$ AU as given by numerical experiment, is to employ $b < -1$. Although some decrease of K_r with r is expected on wave propagation grounds out to 1 AU, the asymptotic predicted behaviour is $K_r = K_0 r^b$ (Skadron & Hollweg 1976; Morfill *et al.* 1976). Therefore it appears that some additional interplanetary source of waves is required in the inner solar system, over and above

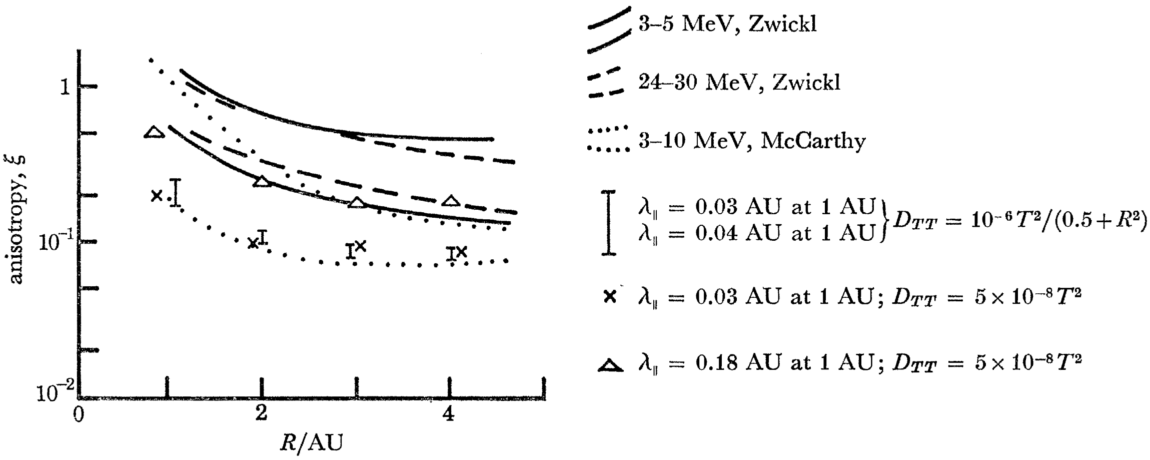

FIGURE 7. Pioneer 10 and 11 data on the solar proton flux anisotropy at the time of maximum particle flux measured by Zwickl & Webber (1977) and McCarthy & O'Gallagher (1976) at different radial distances, together with the predictions of various theoretical models incorporating the parameters shown. Units as for figure 5.

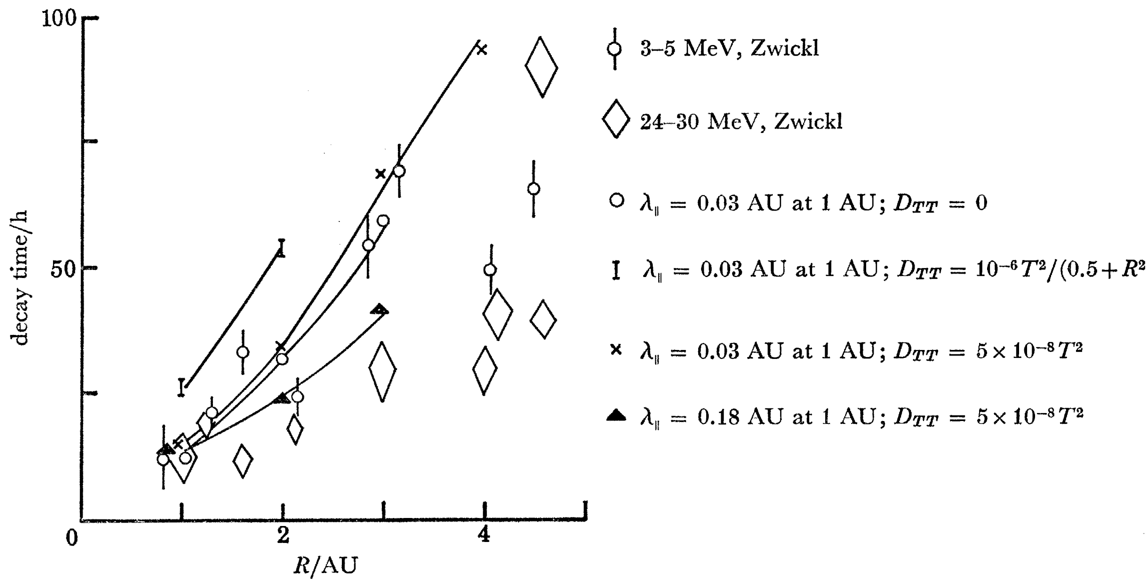

FIGURE 8. Pioneer 10 and 11 data on the decay time constant of solar proton fluxes as given by Zwickl & Webber (1977) for different radial distances, together with the predictions of various theoretical models incorporating the parameters shown. Units as for figure 5.

those propagating from the chromosphere as envisaged in the aforementioned theoretical papers. Hamilton (1976), in a study of Pioneer 10 and 11 data, finds $K \propto r^{0.4}$ at large distances. In order to accommodate the required fall off of K_r at small solar distances and the possible slow increase further out, Cecchini *et al.* (1980) use the radial dependence shown in figure 4 which has a maximum at 1 AU.

Differences in the predictions according to the amount of acceleration present become more pronounced at greater radial distances. Figure 5 shows the intensity time profiles at 2 and 4 AU, all with $K(r)$ given by figure 4. One set of curves adopts the previous values of $\lambda_{\parallel}^{1\,\mathrm{AU}} = 0.03$ AU with D_{TT} either zero or $D_{TT} = 5 \times 10^{-8}\, T^2\, (\mathrm{MeV})^2\, \mathrm{s}^{-1}$ corresponding to the value favoured in §5. An extreme case with $\lambda_{\parallel}^{1\,\mathrm{AU}} = 0.18$ AU, $D_{TT} = 5 \times 10^{-8}\, T^2\, (\mathrm{MeV})^2\, \mathrm{s}^{-1}$ is also computed, this large value of the parallel mean free path being one put forward by Zwickl & Webber (1977) from analysis of Pioneer 10 and 11 data on the solar proton propagation but without reference to the magnetic data. Note from the figure that the acceleration parameter $D_{TT} = 5 \times 10^{-8}\, T^2\, (\mathrm{MeV})^2\, \mathrm{s}^{-1}$ makes little difference to the time profile out to 4 AU but the value 20 times larger significantly enhances the particle population at late times.

Following Hamilton (1976) and Zwickl & Webber (1977) it is useful to use data obtained from prompt solar proton measurements on Pioneers 10 and 11 out to 4 AU on the time to maximum flux, the anisotropy at peak flux and the decay time of the events as further constraints in the fitting of the model solutions to particle data. Figure 6 presents the experimentally measured time of maximum for 3–5 MeV and 24–20 MeV protons determined by Zwickl & Webber and for 11–20 MeV protons determined by Hamilton. Cecchini *et al.* (1980) computations for the parameters used in figure 4 are also shown. While our favoured set ($\lambda_{\parallel}^{1\,\mathrm{AU}} = 0.03$ AU, $D_{TT} = 5 \times 10^{-8}\, T^2$) fits the Zwickl–Webber points well, a longer mean free path seems to be demanded by the Hamilton results. Figure 7 shows the Zwickl–Webber anisotropy at maximum flux data, together with McCarthy & O'Gallagher's (1976) information on 3–10 MeV protons. In this case, the favoured set lies at the low end of the experimental spread while the $\lambda_{\parallel}^{1\,\mathrm{AU}} = 0.18$ AU curve is more central. Finally in figure 7, decay times, measured 24 h after the peak by Zwickl & Webber are plotted, together with the model predictions. While all models and the data agree reasonably well at 1 AU, the experimental decay times tend to be shorter than any model prediction at above 4 AU.

Points which arise from these comparisons are as follows.

1. Experimenters differ rather widely on the time of maximum and anisotropy values for prompt solar events measured at Pioneer 10 and 11.

2. Experimental measurement of the decay time at large distances is very difficult or impossible without adequate azimuthal measurements because a typical event will have a 2–$4\frac{1}{2}$ day solar longitudinal e-folding structure, according to McCracken *et al.* (1971). This will greatly distort the decay time measured on a single spacecraft. Theory only predicts the particle behaviour on a magnetic flux tube that is well connected to the flare site and due to corotation this moves past the spacecraft during the measurement. Hence when the predicted decay times are 5 days or more, discrepancies such as shown in figure 8 are expected.

3. Rotational discontinuities in the magnetic flux tubes that represent a high → low field transition will cause local increases of the particle anisotropy, persisting for several particle cyclotron radii distance (see Moussas *et al.* 1975). Hence the observed anisotropy may tend to be higher than predicted on diffusion theory and the rather low theoretical points in figure 7 may be reconciled in this manner.

Taking into account the problems raised by the above three points, we may conclude that the numerically calculated values of $K_{\parallel}$ and D_{TT} produce particle intensity–time distributions consistent with the known experimental particle fluxes. One important indication is that growth of Alfvénic fluctuations seems to be required between *ca.* 0 AU and *ca.* 1 AU from the Sun.

Many fruitful discussions with my colleagues, Dr S. Cecchini and Dr X. Moussas are acknowledged, together with Science Research Council support for visits of both of these workers to Imperial College. Dr H. Grünwaldt of the Max-Planck-Institut für Physik and Astrophysik, Garching, kindly supplied HEOS-2 plasma data for use in some of the computations while the magnetometer data were obtained through Dr P. C. Hedgecock from the Imperial College experiment on board HEOS-2.

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